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Dust-acoustic waves in a self-gravitating complex plasma with trapped electrons and nonisothermal ions

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Abstract. It is shown that the nonlinear equations governing the dynamics of the large amplitude waves in a self-gravitating unmagnetized collisionless dust-electron-ion plasma admit stationary dust-acoustic shock solutions. Owing to the adiabaticity of dust-charge variation, inclusion of self-gravitation, and to the departure from the so-called Botzmannian electrons and ions to the trapped electrons and nonthermal ions, the dynamics of the nonlinear wave is found to be governed by a new energy-like integral equation.

PACS. 52.35.Sb Solitons; BGK modes – 52.30.Ex Two-fluid and multi-fluid plasmas – 52.35.-g Waves, oscillations, and instabilities in plasmas and intense beams – 52.35.Mw Nonlinear phenomena: waves, wave propagation, and other interactions (including parametric effects, mode coupling, ponderomotive effects, etc.)

1 Introduction

Interests in the field of dusty plasma physics have rapidly been growing because of their versatile applications to laboratory, space and astrophysical plasma environments, viz. asteroid zones, planatery rings, cometary tails, interstellar medium, earth's environments etc. In reality, the charge on the dust grain varies both with space and time due to the electron and ion currents flowing into or out of the dust grain, as well as other processes like secondary emission, photo-emission of electrons. These lead to dust charge fluctuations. The mass of an individual isolated dust grain is typically about 10^6–10^{12} times the ion mass, and hence the mass of dusty plasma is essentially contained in the dust grains. The presence of such fairly massive gives rise a new ultra low-frequency regime for the existence of different types of acoustic modes in dusty plasmas, which do not exist in the usual electron-ion plasmas. One such important mode is the dust-acoustic mode, which is an eigen mode of the dust-electron-ion plasma where the charged dust grains provide the inertia and the pressures of inertialess electrons and ions provide the restoring force [1–19]. Thus, the dust-acoustic waves (DAW) appear on a kinetic level, which have been visualized by the naked eye in several laboratory experiments [20–24]. Typical images of the DAWs reveal that the waves are of large amplitudes and their wave fronts are steepened. It has been found that the harmonic generated nonlinearity gives rise to small amplitude DA solitary waves which are governed by the Kortewg-de-Vries (KdV) equations [20,25], whereas the large amplitude DA solitary waves are shown to exist in the steady state only [17,26]. It has been suggested that the high-speed streaming particles excite various kind of nonlinear waves in space [27,28]. It may be noted that the existence of dust acoustic wave on a slow time scale was first investigated by Rao et al. [1] who showed the formation of rarefactive type soliton in a simple dusty plasma. Several authors have investigated similar phenomena by considering dust charge dynamics and using reductive perturbation method [29].

The gravitational effects become important when the sizes of the dust grains become considerable. For too small grains, trapping by magnetic field lines dominates much as it does for other plasma particles. Too large grains on the other hand follow gravitationally bound orbits, without being distracted from them by electromagnetic forces. In plasmas like protostellar clouds, there may be competition between the gravitational self-attraction and electrostatic repulsion between the grains. When self-gravitational interaction due to the behaviour dust component is included, dusty plasmas are subject to macroscopic instabilities of the Jeans type [30–33]. Physically, the Jeans instability of a massive system aries due to the purely attractive gravitational force, which is unlike the electromagnetic force.

Shock waves often arise in nature because of a balance between wave-breaking nonlinear and the combined influence of dispersive and wave-damping dissipative forces.

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When the dissipation dominates over the dispersion, shock front exhibits monotonic structure, whereas in the opposite case, the shock transition is of the oscillatory type. Collisional and collisionless shock waves can appear in acoustic wave propagation because of friction between the particles and wave particle interaction. It has been found that the nonadiabaticity of dust charge variation provides an alternate physical mechanism causing dissipation and as a consequence this gives rise to shocks for which both monotonic and oscillatory structures are possible.

It has been confirmed by computer simulation and experiments that when streaming particles be injected in plasma, it is found that they often evolve towards a coherent trapped particle state, instead of developing into a turbulent one [34,35]. Also, in the formation of double layers [36] and computer simulation [37], the onset of an electron trapping is seen. It is beyond doubt that the inclusion of trapped electrons or ions or both give rise nonlinear phenomena of waves. However, it has recently been found that the electron and ion distributions play a crucial role in characterizing the physics of nonlinear waves [28,38,39]. They offer considerable increase in reachness and variety of wave motion which can exist in plasmas and further influence the conditions required for the formation of these structures. Moreover, it is also well known that the electron and ion distributions can be significantly modified in the presence of large amplitude waves [40]. Also, the inclusion of thermal effects also affects the nature of wave-particle interaction and possibility of having nonisothermal electron distribution in the potential well instead of the usual Boltzmann law is often invoked. The simultaneous presence of trapped and free electrons can significantly modify the wave propagation characteristics in collisionless plasmas.

It has been said in reference [28] that when the amplitude of nonlinear wave becomes large, electrons are trapped in the potential trough. As a matter of fact, the trapping of electrons is not a question of strength of the amplitude. Even for small amplitudes trapping can occur and contribute, violating thereby linear wave analysis [41]. Thus inclusion of nonisothermal electrons in the nonlinear wave phenomena is indispensable to consider nonlinear wave structures. Nejoh [28] investigated large amplitude electrostatic ion waves by considering dust charge fluctuation and trapped electrons. It was shown there that the existence of the nonlinear ion waves depends on the Mach number as well as the ion to electron temperature ratio. Recently, El-Labany et al. [38,39] derived small amplitude DA solitons and double layers as well as stationary modes with considering the dust charge dynamics together with trapped electrons/ions. In reference [39] it was shown that the two-ion temperature provides the possibility for the coexistence of rarefactive and compressive DA solitary structures and double layers. These investigations, however, where the gravitational force is neglected are valid only in that plasma regime in which the electrostatic force is much greater than the gravitational one. In our present work we have focused our attention to those space and astrophysical plasma regimes where the gravitational action is comparable to or greater than that of electrostatic one [14,15], and have investigated the nonlinear structures of large amplitude DA waves in an unmagnetized self-gravitating warm dusty plasma by incorporating the effects of trapped electrons as well as the nonisothermal positive ions with finite temperatures. Although, a number of papers considered the effect of trapped electrons/ions, dust charge fluctuations, dust temperatures etc., but none has so far considered the effects of self-gravitation, trapped electrons, nonthermal ions, dust temperature, dust charge fluctuations all together.

Our manuscript is organized as follows: in Section 2, we present the hydrodynamic equations for the dust fluid and density distributions for the trapped electrons and nonthermal ions. Section 3 is devoted to derive a set of nonlinear differential equations describing the dynamics of the large amplitude dust waves. In Section 4, the energy-like equation is derived by the Hamiltonian formulation in the adiabaticity of dust-charge variation and lastly, the concluding session is completed in Section 5.

2 Basic equations

We consider an unmagnetized self-gravitating collisionless plasma consisting of extremely massive and highly negatively charged warm dust grains of equal radii, positively charged nonisothermal ions together with free and trapped electrons. Thus, at equilibrium the overall charge neutrality condition reads

$$n_{i0} = n_{e0} + Z_{d0} n_{d0} \tag{1}$$

where n_{j0} with $j = e, i, d$ respectively stand for unperturbed number densities for electron, ion and dust, and Z_{d0} is the number of electrons residing on the dust grain surface. The nonlinear dynamics of the DA waves in such a dusty plasma is governed by

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0 \tag{2}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} + \frac{\sigma_d}{n_d}\frac{\partial P}{\partial x} - Z_d \frac{\partial \phi_E}{\partial x} + \frac{\partial \phi_G}{\partial x} = 0 \tag{3}$$

$$\frac{\partial P}{\partial t} + u_d \frac{\partial P}{\partial x} + \gamma P \frac{\partial u_d}{\partial x} = 0 \tag{4}$$

$$\frac{\partial^2 \phi_E}{\partial x^2} = Z_d n_d + n_e - n_i \tag{5}$$

$$\frac{\partial^2 \phi_G}{\partial x^2} = 4\pi G m_d n_d \tag{6}$$

where n_d and u_d are the dust number density and dust fluid velocity normalized to $Z_{d0} n_{d0}$ and C_d(dust acoustic speed) $= \sqrt{Z_{d0} T_{eff}/m_d}$ with $T_{eff} = Z_{d0} n_{d0} T_i T_e/(n_{i0} T_e + n_{e0} T_i)$; T_i, T_e being the ion and electron temperature and m_d the dust mass; $\phi_E(\phi_G)$ is the electrostatic (gravitational) potential normalized to $T_{eff}/e(C_d^2)$ with e being the elementary charge; P is the dust pressure normalized to $n_{d0} T_d$; $\gamma = (2+N)/N$ with N being the number of degrees of freedom ($N = 1$

for one dimensional case and $N = 3$ for three dimensional case). Also $\sigma_d = T_d/(Z_{d0}T_{eff})$ and G the universal constant of gravitation. Moreover, the space coordinate (x) and time (t) are normalized to the Debye length $\lambda_{Dd} = \sqrt{T_{eff}/4\pi Z_{d0}n_{d0}e^2}$ and the inverse of the dust plasma frequency $\omega_{pd} = \sqrt{4\pi Z_{d0}^2 e^2 n_{d0}/m_d}$.

In the dynamical system, some of the electrons are attached to form charged dust grains and some remaining are bounded back and forth in the potential well loosing energy continuously and thereby trapped. Schamel [42] presented a new method for constructing a smooth distribution for the trapped particles. The validity of such a distribution function for a magnetized plasma is discussed by Bujarbarua et al. [43]. The number density of the nonisothermal electrons in normalized form is obtained by taking first moment of Schamel's distribution function as:

$$n_e(\phi_E) = \frac{n_{e0}}{n_{d0}Z_{d0}}\left[\exp(\Gamma)\mathrm{erf}c(\sqrt{\Gamma}) + \frac{1}{\sqrt{|\beta_h|}}\right.$$
$$\left.\times\begin{cases}\exp(\Gamma\beta_h)\mathrm{erf}(\sqrt{\Gamma\beta_h}) & \beta_h \geq 0\\ \dfrac{2}{\sqrt{\pi}}\exp(\Gamma\beta_h)\int_0^{\sqrt{-\Gamma\beta_h}}\exp(t^2)dt & \beta_h < 0\end{cases}\right] \quad (7)$$

where $\Gamma = e\phi_E/T_e$, $\beta_h = T_e/T_t$ with T_t the trapped electron temperature. The electron density includes three types of distributions namely, (i) The Maxwellian where $\beta_h \to 1$ (ii) the flat topped one where $\beta_h = 0$ and (iii) a hole in the trapped region representing a vortex type distribution where $\beta_h < 0$. Now expanding equation (6) for small arguments by Taylor series we write the expression for n_e [38] as

$$n_e = \nu[\exp(s\sigma_i\phi_E) - G(s\sigma_i\phi_E)] \quad (8)$$

where

$$G(x) = \sum_{k=1}^{n}\left[2^{k+1}b_k x^{(2k+1)/2}/\prod(2k+1)\right] \quad (9)$$

with $b_k = (1-\beta_h^k)/\sqrt{\pi}$. Thus the isothermality and nonisothermality of electrons are described by imposing $b_k = 0$ and $0 < b_k < 1/\sqrt{\pi}$ respectively. On the other hand, the ion density distribution is assumed to describe by the following

$$n_i = \mu\left(1 + \beta\phi_E + \beta\phi_E^2\right)\exp(-s\phi_E) \quad (10)$$

where $\beta = 4\eta/(1+3\eta)$ with η being a parameter determining the number of nonthermal ions present in our plasma model. Mendoza et al. [12] used such type of distribution for nonthermal ions in a dusty plasma and they found that nonthermal ions change the nature of the DA solitary waves and support the co-existence of large amplitude compressive and rarefactive solitary waves. Also, Mamun [15] using the same distribution in a self-gravitating dusty plasma found that the effects of nonthermal ions play a role in stabilizing the electrostatic modes and counter the gravitational condensation of the dust grains. In equations (8) and (10) the following dimensionless variables are used:

$$\nu = n_{e0}/Z_{d0}n_{d0}, \quad \mu = n_{i0}/Z_{d0}n_{d0},$$
$$\sigma_i = T_i/T_e, \qquad s = 1/(\mu + \sigma_i\nu) \quad (11)$$

where ν and μ are connected through the relation

$$\mu - \nu = 1. \quad (12)$$

We note that Z_d appearing in equations (3) and (5) are not constant but varies with space and time and so the dust-charge dynamics is governed by the following normalized equations:

$$\left(\frac{\partial Z_d}{\partial t} + u_d\frac{\partial Z_d}{\partial x}\right) = -\frac{\tau_d}{Z_{d0}e}(I_e + I_i) \quad (13)$$

where $\tau_d = \omega_{pd}^{-1}$ is the hydrodynamic time scale. Assume that the streaming velocities of the electrons and the ions are much smaller than their thermal velocities, the electron and ion currents (I_e, I_i) arriving on the surface of the spherical dust grains with radius r due to thermal fluxes of electrons and ions are given as

$$I_e = -\pi r^2 e\sqrt{\frac{8T_e}{\pi m_e}}n_e\exp(s\sigma_i\Psi) \quad (14)$$

$$I_i = \pi r^2 e\sqrt{\frac{8T_i}{\pi m_i}}n_i(1 - s\Psi) \quad (15)$$

where $\Psi = e\Phi/T_{eff}$; $\Phi = -Z_d e/C$ being the dust grain surface potential relative to the plasma potential ϕ_E and $C = r\exp(-r/\lambda_D)$ the capacitance. The applicability of the charging equation (13) for the case of strong nonlinearity and particle trapping is discussed in the Appendix B. Also, the equilibrium dust surface potential $\Psi_0(=\Psi/Z_d)$ can be obtained from the equilibrium current balance equation $dZ_d/dt = 0$ as:

$$\Psi_0 = \ln\left[\alpha\delta(1 - s\Psi_0)\right]/s\sigma_i \quad (16)$$

where

$$\alpha = \sqrt{\sigma_i/\mu_i};\ \mu_i = m_i/m_e \approx 1836;\ \delta = n_{i0}/n_{e0}. \quad (17)$$

3 Nonlinear dust-acoustic wave

In order to study the dynamics of the large amplitude DA waves in presence of nonadiabatic dust-charge fluctuations, self-gravitation, trapped electrons and nonisothermal ions we assume that all the variables in equations (2–6, 13) depend only on a single one $\xi = x - Mt$, where M is the Mach number (the velocity of the moving frame normalized to the DA speed C_d). In this stationary frame these equations become

$$-M\frac{\partial n_d}{\partial \xi}+\frac{\partial}{\partial \xi}(n_d u_d)=0 \tag{18}$$

$$-M\frac{\partial u_d}{\partial \xi}+u_d\frac{\partial u_d}{\partial \xi}+\frac{\sigma_d}{n_d}\frac{\partial P}{\partial \xi}-Z_d\frac{\partial \phi_E}{\partial \xi}+\frac{\partial \phi_G}{\partial \xi}=0 \tag{19}$$

$$-M\frac{\partial P}{\partial \xi}+u_d\frac{\partial P}{\partial \xi}+3P\frac{\partial u_d}{\partial \xi}=0 \tag{20}$$

$$\frac{\partial^2 \phi_E}{\partial \xi^2}=Z_d n_d+n_e-n_i \tag{21}$$

$$\frac{\partial^2 \phi_G}{\partial \xi^2}=4\pi G m_d n_d \tag{22}$$

$$M\frac{\partial Z_d}{\partial \xi}=\frac{n_d}{\omega_{pd} Z_{d0} e}\left(I_e+I_i\right) \tag{23}$$

where we have taken $\gamma = 3$ for $N = 1$ (for isothermal case $\gamma = 1$ and for adiabatic $\gamma = 3$).

Now under the appropriate boundary conditions, viz. $n_d \to 1, u_d \to 0, P \to 1$ at $\xi \to \pm\infty$, equations (18) and (20) can be integrated to yield

$$u_d = M(1-1/n_d) \tag{24}$$

$$P = n_d^3. \tag{25}$$

We eliminate n_d from equations (19) and (24) and multiplying the resulting equations so obtained by 2 and then subtract from the equation (20) multiplied by σ_d/M to obtain

$$\frac{dn_d}{d\xi}=\frac{n_d^3}{3\sigma_d n_d^4-M^2}\left[Z_d\frac{d\phi_E}{d\xi}-\frac{d\phi_G}{d\xi}\right]. \tag{26}$$

Now, further integration of equation (26) with respect to ξ and use of the boundary conditions $P \to 1$, $u_d \to 0$, $\phi_E, \phi_G \to 0$ as $\xi \to \pm\infty$ yield a biquadratic equation for n_d as

$$3\sigma_d n_d^4-(3\sigma_d+M^2+2V)n_d^2+M^2=0 \tag{27}$$

where

$$V=V_d(\phi_E)-\phi_G, V_d(\phi_E)=\int_0^{\phi_E} Z_d d\phi_E. \tag{28}$$

Therefore the solutions of equation (27) are given by

$$n_d = \left[\frac{(3\sigma_d+M^2+2V)\pm\sqrt{(3\sigma_d+M^2+2V)^2-12\sigma_d M^2}}{6\sigma_d}\right]^{1/2}. \tag{29}$$

For the real value of n_d we must have

$$\phi_G \le \frac{1}{2}(M^2+3\sigma_d)-M\sqrt{3\sigma_d}+V_d(\phi_E). \tag{30}$$

Thus, from equation (29) we find that as $\phi_G \to (\phi_G)_{max}$, $n_d \to \sqrt{M/\sqrt{3\sigma_d}}$. Which shows that the dust density increases as the Mach number increases for fixed σ_d. Such dust condensations may initiate the gravitational collapse in interstellar dust cloud leading to star formation [43]. On the other hand, as the dust density increases it follows from equation (23) that u_d approaches the corresponding value of M, i.e., the dust fluid velocity will tend to move with the phase velocity, an effect that will cause the electrons and ions to accumulate in the dense region.

Now letting $Z_d = 1 + Z_{d1}$ and using the expressions (8–10), (14–16) we obtain from equations (20–22) and (25) the following system of equations with higher nonlinearities, which govern the dynamics of the large amplitude dust-acoustic waves in our plasma model

$$\frac{d^2\phi_E}{d\xi^2}=(1+Z_{d1})n_d+\nu[\exp(s\sigma_i\phi_E)-G(s\sigma_i\phi_E)] - \mu(1+\beta\phi_E+\beta\phi_E^2)\exp(-s\phi_E) \tag{31}$$

$$\frac{d^2\phi_G}{d\xi^2}=Z_{d0}\zeta^2 n_d \tag{32}$$

$$\begin{aligned}\frac{dZ_{d1}}{d\xi}=&-\frac{\beta_{diss}/(Z_{d0}n_{d0})^2}{M\omega_{pd}}[\exp(s\sigma_i\psi_0 Z_{d1})\\ &\times(\exp(s\sigma_i\phi_E)-G(s\sigma_i\phi_E))\\ &-(1+\beta\phi_E+\beta\phi_E^2)(1-s\psi_0 Z_{d1}/(1-s\psi_0))\\ &\times\exp(-s\phi_E)]\end{aligned} \tag{33}$$

$$\frac{dn_d}{d\xi}=\frac{n_d^3}{3\sigma_d n_d^4-M^2}\left[(1+Z_{d1})\frac{d\phi_E}{d\xi}-\frac{d\phi_G}{d\xi}\right] \tag{34}$$

where $\zeta = \omega_{jd}/\omega_{pd}$ is the Jeans to plasma frequency ratio with $\omega_{jd} = \sqrt{4\pi G m_d n_{d0}}$ being the Jeans frequency, and $\beta_{diss} = (|I_{i0}|/e)(n_{d0}/n_{i0})$ represents a dissipation rate that is similar to collisional dissipation. It can lead to damping or generation of waves depending on the circumstances.

We numerically integrate the equations (31–34) by the fifth order Runge-Kutta-Fehlberg method starting with the small perturbation 0.02 each for ϕ_E, ϕ_G, n_d from the equilibrium at $\xi = 0$. Since the right hand expressions of equations (31–34) are explicitly free from ξ we can take $\xi = 0$ as the upstream point and integrate upto large possible positive value of ξ. The simulation parameters that are typical for the photoassociation regions separating H II regions from dense molecular clouds [48] taken as follows: $T_e = 30$ K, $T_i = 10$ K, $T_d = 1$ K, $n_{i0} = 2\times10^{-3}$ cm^{-3}, $n_{d0} = 5\times10^{-7}$ cm^{-3}, $Z_{d0} = 2000$, $m_d = 10^{-11}$ g, $a = 10^{-4}$ cm. It is found that the perturbations develop into shocks provided the Mach number has the extreme values depending on the parameters σ_i, β, β_h. Figures 1–7 depict the behaviour of electrostatic potential, dust number density and dust charge for different values of mass to charge ratio $r_{jp} = \omega_{jd}/\omega_{pd}$ (Figs. 1–4) and for different β, β_h (Figs. 5–7). We find that as the ratio $r_{jp} < 1$ increases the shock wave transits from oscillatory to monotonic one. Figures 2, 3 show the density profiles in the shocks, where the wave steepening turns into a monotonic shock of the classic shape. The dust number density and dust charge number enhance for $r_{jp} = 0.24, 0.81$ (Figs. 3 and 4) when the self-gravitational influence is strong. In Figure 5 for $r_{jp} = 0.81$, as the percentage of fast particles

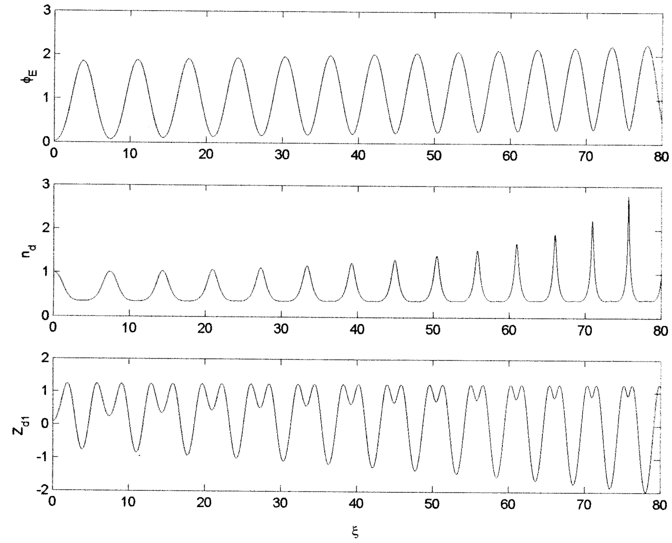

Fig. 1. The electrostatic potential (ϕ_E), dust number density (n_d) and the dust charge (Z_{d1}) profiles (oscillatory) in a shock wave at fixed $Z_{d0} = 10^3$, $r_{jp} = 5.38 \times 10^{-4}$, $\sigma_d = 1.79 \times 10^{-3}$, $\sigma_i = 0.33$, $\beta = 0.1$, $\beta_h = 0.4$, $M = 0.9$ showing that the subsonic flow can exist.

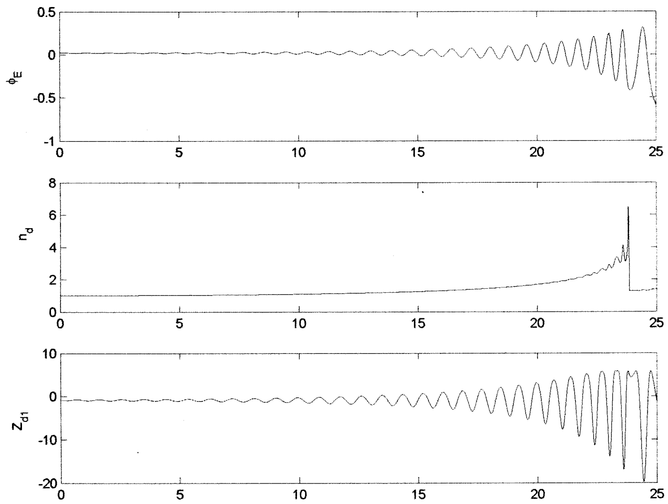

Fig. 2. The same as in Figure 1 with $Z_{d0} = 200$, $r_{jp} = 2.69 \times 10^{-2}$, $\sigma_d = 4.47 \times 10^{-2}$, $\sigma_i = 0.33$, $M = 10.0$ (other parameter values remain the same as Fig. 1), showing that the supersonic flow exists and the dust density transits form oscillatory to the monotonic shock on the downstream side.

(β) = 0.5 increases and the free to trapped electron temperature ratio ($\beta_h = 0.2$) decreases the ϕ_E, Z_d show the behaviour of growing oscillations and the dust number density remains constant on the far downstream side. Figure 6 shows that for fixed $r_{jp} = 0.81, \beta = 0.5$, as β_h increases from 0.2 to 0.4 n_d exploids to nearly $1.5 \times 10^4\ \mathrm{cm}^{-3}$ and Z_d to -1.8×10^5 on the downstream. From Figure 7 we find that for fixed $r_{jp} = 0.81, \beta_h = 0.2$, as β increases from 0.5 to 0.7 the dust density again steepens and turns into monotonic shock on the downstream.

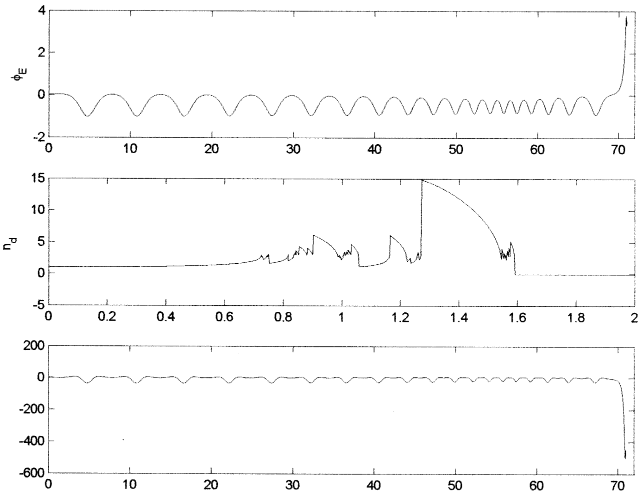

Fig. 3. The same as in Figure 1 with $r_{jp} = 0.24$, $M = 3.0$ (other parameter values remain the same as Fig. 2), showing that the wave steepens and the oscillations behind the shock increase and the dust density becomes large in $1.2 < \xi < 1.4$.

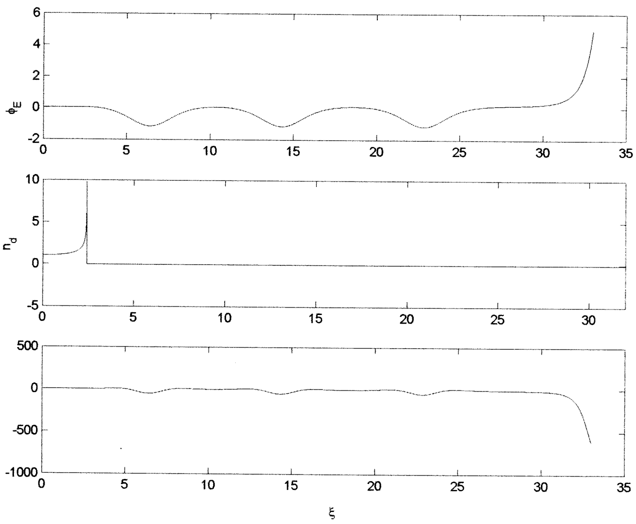

Fig. 4. The same variations as Figure 1, but for $r_{jp} = 0.81$, $M = 30.0$ (other parameter values remain the same as Fig. 2), showing the monotonic shock for the density profile and the magnitude of the dustcharge becomes large on the downstream.

4 Adiabatic dust-charge variation: energy-like equation

However, self-gravitation is important for astrophysical scenarios, there may be the case in space where the dust charging time scale τ_{ch} is of the order of $10^{-6} - 10^{-4}$ s [25], so that on the hydrodynamic time scale (τ_d), the dust charge can quickly reach local equilibrium at which the electron and ion currents I_e and I_i give $I_e + I_i \approx 0$, so that we have

$$\exp(s\sigma_i \Psi) = \frac{\alpha\delta(1 - s\Psi)\exp(-s\phi_E)(1 + \beta\phi_E + \beta\phi_E^2)}{\exp(s\sigma_i\phi_E) - G(s\sigma_i\phi_E)}. \tag{35}$$

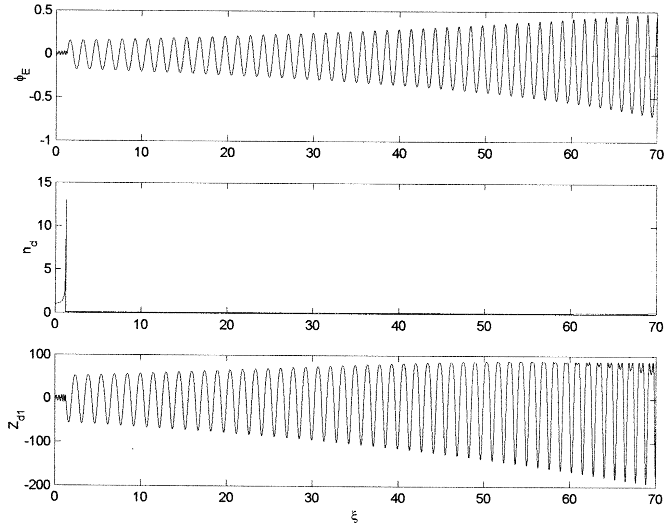

Fig. 5. The same profiles as Figure 1, but for different $\beta = 0.5$, $\beta_h = 0.2$, $M = 15.0$ (other parameter values remain the same as Fig. 4), showing that the ϕ_E, Z_{d1} become oscillatory and the dust density increases on the upstream side and the magnitude of the dustcharge decreases on the downstream.

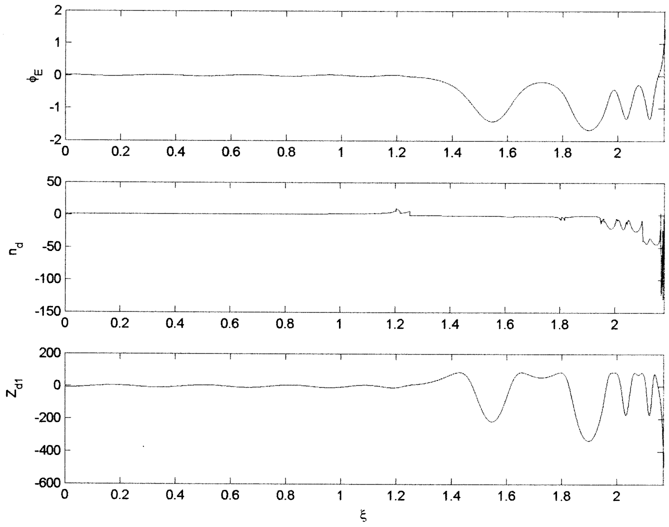

Fig. 6. The same profiles as Figure 1, but for different $\beta_h = 0.4$ (other parameter values remain the same as Fig. 5), showing the enhancement of dust number density and the magnitude of the dust charge. n_d, Z_{d1} exploid to 1.5×10^4 and -1.8×10^5 respectively.

Substitutions of the normalized number densities of electrons, ions and dusts as well as dust charge number into the Poisson equations (21) and (22) lead to the following coupled system

$$\frac{\partial^2 \phi_G}{\partial \xi^2} = \eta^2 / U(\phi_E, \phi_G, M) \tag{36}$$

$$\begin{aligned}\frac{\partial^2 \phi_E}{\partial \xi^2} &= Z_d(\phi_E)/U(\phi_E, \phi_G) \\ &\quad + \nu\left[\exp(s\sigma_i \phi_E) - G(s\sigma_i \phi_E)\right] \\ &\quad - \mu\left(1 + \beta\phi_E + \beta\phi_E^2\right)\exp(-s\phi_E)\end{aligned} \tag{37}$$

where

$$U(\phi_E, \phi_G) = \sqrt{1 + \frac{2(V_d - \phi_G)}{M^2 - 3\sigma_d}}, \quad \eta^2 = Z_{d0}\frac{\omega_{jd}^2}{\omega_{pd}^2} \tag{38}$$

In order to find V_d it is necessary to find an explicit expression for Z_d which is obtained by the perturbation technique upto the cubic order of ϕ_E as:

$$\begin{aligned}Z_d(\phi_E) = 1 + \Bigg(&\gamma_1\phi_E + \frac{3}{2}\gamma_2\phi_E^{3/2} \\ &+ 2\gamma_3\phi_E^2 + \frac{5}{2}\gamma_4\phi_E^{5/2} + 3\gamma_5\phi_E^3\Bigg) \Bigg/ \Psi_0.\end{aligned} \tag{39}$$

The expressions for γ's are given in the Appendix A.

The system of equations (36) and (37) being a pair of ordinary second order differential equations with strong nonlinearities, describe the stationary modes of our dusty plasma depending on the plasma parameters viz., $\beta, \beta_h, \sigma_i, \sigma_d, Z_d, \delta$ etc. The higher nonlinearity feature and structure of these equations prevent any hope for exact solution. However, it is possible to construct an exact conservation law by introducing the variables:

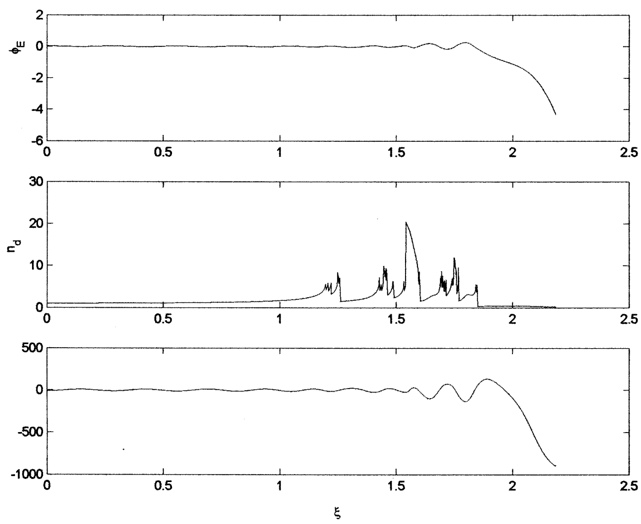

Fig. 7. The same profiles as Figure 1, but for different $\beta = 0.7$ (other parameter values remain the same as Fig. 5), showing that for n_d the oscillations behind the monotonic shock increase and the magnitude of the dust charge decrease on the downstream.

$$\bar{\phi}_G = i\phi_G, \quad \bar{\phi}_E = \eta^2\phi_E, \quad \bar{\xi} = \xi\eta \tag{40}$$

where $\eta^2 = Z_{d0}\omega_{jd}^2/\omega_{pd}^2$, and the system then takes the following Hamiltonian form

$$\frac{d^2\bar{\phi}_G}{d\bar{\xi}^2} = -\frac{\partial W}{\partial\bar{\phi}_G}, \quad \frac{d^2\bar{\phi}_E}{d\bar{\xi}^2} = -\frac{\partial W}{\partial\bar{\phi}_E} \tag{41}$$

where $W(\phi_E, \phi_G)$ is the pseudopotential. From equation (41) one can immediately obtain the energy-like integral equation as

$$\frac{1}{2}\left(\frac{d\bar{\phi}_E}{d\xi}\right)^2+\frac{1}{2}\left(\frac{d\bar{\phi}_G}{d\xi}\right)^2+W(\bar{\phi}_E,\bar{\phi}_G)=0 \quad (42)$$

which can now be converted into in terms of the original variables as

$$I\equiv-\frac{1}{2}\left(\frac{d\phi_E}{d\xi}\right)^2+\frac{1}{2}\left(\frac{d\phi_G}{d\xi}\right)^2+W(\phi_E,\phi_G)=0 \quad (43)$$

where the potential W is given by

$$\begin{aligned}W(\phi_E,\phi_G)&=(M^2-3\sigma_d)\left[1-U(\phi_E,\phi_G)\right]\\&+\frac{\nu}{s\sigma_i}\left[1-\exp(s\sigma_i\phi_E)+\int_0^{\phi_E}G(s\sigma_i\phi_E)d\phi_E\right]\\&+\frac{\mu}{s}\Big[1-\exp(-s\phi_E)-\beta\{\phi_E^2+(1+2/s)\\&\times(\phi_E+1/s)\}\exp(-s\phi_E)+\frac{\beta}{s}(1+2/s)\Big].\end{aligned} \quad (44)$$

Equation (43) is our new form of energy-like integral equation with the new pseudopotential W with strong nonlinearities. The second term in equation (43) and the term ϕ_G in W arise due to the action of the self-gravitational force. Also, the terms proportional to β in W are due to the presence of fast particles in the plasma. In absence of nonthermal ions and the self-gravitation, i.e., simply for $\beta=0$, $\phi_G=d\phi_G/d\xi=0$ one can recover the energy equation as in reference [38]. It is to be mentioned that in the derivation of equation (43) we have considered the adiabatic dust charge variation, i.e. when $\tau_{ch}/\tau_d=0$ and no such pseudopotential can exist when nonadiabaticity of dust charge variation is taken into account. For $\omega_{jd}^2\neq0$, equations (36) and (37) does not have any steady state solution, whereas for $\omega_{jd}^2=0$, there exists a steady state solution, viz. $\phi_E=0=Z_{d1},n_d=1$. Unfortunately, the level surfaces of the integral I are never convex which prevents any hope for its use as a Lyapunov function for the stability analysis of the system at the critical points. However, for $\omega_{jd}^2=0$, some further insight about the stability of the system can be obtained by linearizing the system of equations (36) and (37) around the steady state solution, which is also rather difficult to do so analytically, since the right side of equation (37) contains various typical powers of ϕ_E. Hence we restrict here to small ϕ_E, i.e. keeping the first order terms for ϕ_E and taking $d\phi_E/d\xi=d\phi_G/d\xi=0$ we obtain the following two possible homogeneous solutions for arbitrary values of ϕ_{G0} and ϕ_{E0} respectively:

$$\phi_{E0}=-1/\left[\frac{\gamma_1}{\psi_0}+(s\sigma_i-\beta+s)\left(1-\frac{2\phi_{G0}}{M^2-3\sigma_d}\right)\right] \quad (45)$$

$$\phi_{G0}=\frac{[1+(\gamma_1/\psi_0+s\sigma_i-\beta+s)\phi_{E0}](M^2-3\sigma_d)}{2(s\sigma_i-\beta+s)\phi_{E0}}. \quad (46)$$

The linear stability analysis is assessed by assuming

$$\phi_E=\phi_{E0}+\alpha_0\exp(k_c\xi),\ \phi_G=\phi_{G0}+\beta_0\exp(k_c\xi) \quad (47)$$

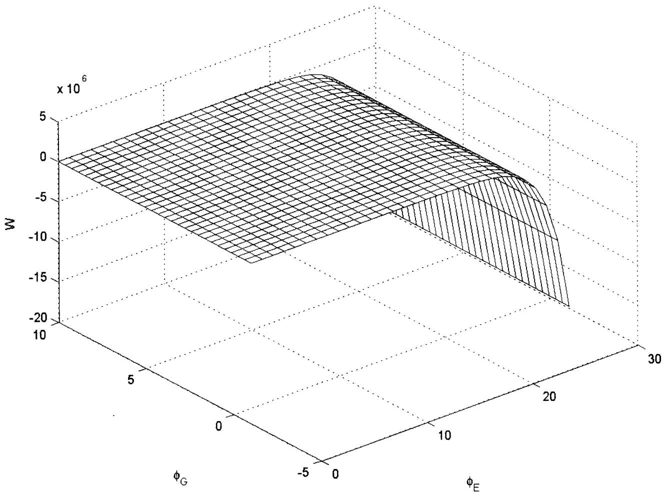

Fig. 8. The three-dimensional view of the pseudopotential $W(\phi_E,\phi_G)$ for $z_{d0}=10^4, r_{jp}=0.48, \beta=0.1, \beta_h=0.5$.

for constants α_0, β_0, k_c. The eigen values thus obtained are given by

$$\begin{aligned}k_c^2=o,&[\gamma_1/\psi_0+\nu(s\sigma_i-\beta+s)]\\&+(1+\gamma_1\phi_{E0}/\psi_0)\frac{2\alpha_0}{\beta_0(M^2-3\sigma_d)}.\end{aligned} \quad (48)$$

Which shows the existence of four eigen values around the homogeneous equilibria of the linearized motion. For purely oscillatory motion (neutral stability) we require $k_c^2\leq0$ with the case $k_c=0$ corresponding to the marginal stability. A three-dimensional view of the pseudopotential is shown in the Figure 8.

5 Conclusions

In our above analysis we have considered the stationary propagation of fully nonlinear arbitrary amplitude dust-acoustic waves in an unmagnetized collisionless self-gravitating plasma. We have employed nonthermally distributed ions and trapped electrons and hydrodynamic descriptions for the warm dust fluid to derive a set of wave equations with strong nonlinearities, which are then ultimately integrated to solve numerically. An analytic solution for the dust density is derived and its limit of validity is reported. It is found that as the gravitational potential gets maximized (i.e. $\phi_G\to\phi_{Gmax}$), the dust density becomes proportional to the Mach number (i.e. $n_d\to\sqrt{M/\sqrt{3\sigma_d}}$). Accordingly, the dust condensation becomes very intense at $M=M_{max}$, leading to the initiatiation of gravitational collapse. When the dissipation caused by the self-gravitation is weak, the shock front exhibits oscillatory nature, while for the stronger dissipation, the shock transition is monotonic. The parameters β and β_h change the situation significantly. The former one plays also a role to transit the growing oscillations into monotonic shocks of the classic shape and the latter enhances the dust number density and the dust charge number.

$$\gamma_1 = -\frac{(1+\sigma_i-\beta/s)(1-s\Psi_0)}{1+\sigma_i(1-s\Psi_0)}, \qquad \gamma_2 = \frac{4}{3}\frac{(s\sigma_i^3)^{1/2}(1-s\Psi_0)}{1+\sigma_i(1-s\Psi_0)}b_1$$

$$\gamma_3 = \frac{s(1-s\Psi_0)}{2\left(1+\sigma_i(1-s\Psi_0)\right)^3}\left[-(1+\sigma_i)^2 + \frac{2\beta}{s}\left(1+\sigma_i(1-s\Psi_0)\right)\left(\frac{1+\sigma_i}{s}-(1-s)\sigma_i\Psi_0\right)\right]$$

$$\gamma_4 = -\frac{1}{(1+\sigma_i(1-s\Psi_0))}\left[(1-s\Psi_0)\sigma_i\left(s\sigma_i\gamma_2 - \frac{4}{3}b_1\gamma_1(s\sigma_i)^{3/2} - \frac{8}{15}b_2(s\sigma_i)^{5/2} + s\sigma_i\gamma_1\gamma_2\right) + (\beta-s)\gamma_2\right]$$

$$\gamma_5 = -\frac{1}{18\left(1+\sigma_i(1-s\Psi_0)\right)}\left[(1-s\Psi_0)\sigma_i\left\{s^2\sigma_i^2(1+3\gamma_1+\gamma^3) + 3s\sigma_i(\gamma_1^2+\gamma_2^2+2\gamma_1\gamma_3) + 6\left(\gamma_3 - \frac{4}{3}\gamma_2 b_1(s\sigma_i)^{3/2}\right)\right\}q \right.$$
$$\left. + (1-s\Psi_0)\left\{6(1-\gamma_1)\beta - 3(1-2\gamma_1)\beta s + s^2(1-3\gamma_1)\right\} + \gamma_3(\beta-s)\right]$$

In the adiabatic dust-charge variation the dynamics of the DA waves is governed by a coupled nonlinear second order differential equations for both electrostatic and gravitational potentials, which in turn, give the exact constant of motion. The linear stability analysis is assessed for a particular case where $\omega_{jd}^2 = 0$ and for small ϕ_E. It is found that the nonlinear DA wave is at the marginal stability point. It is to be mentioned that the trapping of ions which is the largest effect caused by the ion-neutral collisions is important in the nonlinear analysis. However, this is not the case studied here, and this effect will be considered in our next communication. The existence of such dust-acoustic shock waves can be relevant in the study of astrophysical and laboratory dusty plasmas.

The authors are grateful to the referees for their valuable suggestions.

Appendix A

Here we give the expressions for the γ's

see equations above.

Appendix B

Expressions (14) and (15) were derived on the basis of the orbit-motion-limited (OML) theory [40]. This is the traditional method used to determine the charge acquired by a dust particle in a plasma. Actually, this theory has its origin in the probe theory for the case of an infinitely large sheath [49]. In this theory it was assumed that some of the ions in any given energy range hit the probe at grazing incidence. This may not be the case. However, the limiting orbit, i.e. that of ion which is just captured by the probe, may graze a mathematical surface that has a greater radius than that of the probe. Thus we have to consider the concept of an absorption radius or effective potential barrier, which effectively replaces the radius of the probe [50], and the absorption radius is not the same as the effective radius of the "target area". If all the mathematical spherical surfaces outside the probe are grazed by ions with a certain energy range then we have

$$r(1-V/V_0)^{1/2} > r_p(1-V_p/V_0)^{1/2}$$

or,

$$\frac{V_0-V}{V_0-V_p} > \left(\frac{r_p}{r}\right) \tag{49}$$

where $h_p = r_p(1-V_p/V_0)^{1/2}$ is the effective radius of the "target area" presented by the dust particle, eV_0 is the initial energy of ion and V_p the velocity at the probe surface. If this condition is to hold for all the ions, including those with small initial energies then

$$\frac{V}{V_p} > \left(\frac{r_p}{r}\right). \tag{50}$$

If the OML theory is to valid, the potential distribution must satisfy the condition (50), otherwise we have to consider absorption radii (which are different for ions of different energies). It has been shown that the OML theory is in fact never valid for Maxwellian plasmas, at least when $T_i \leq T_e$ [51].

More recently, it has been emerged that the expressions (14, 15) are correct only in particular limits, viz. when the size of the dust grains are small compared to the typical Debye length (λ_D) of the system and, above all, only for isolated dust grains ($r < \lambda_D < d$, d being the average intergrain spacing). The validity of the OML theory has been extensively scrutinized by Allen et al. [41]. It was found that as long as the grain size is much smaller than the Debye length and for typical values of the temperature ratio σ_i, OML theory remains valid [42]. This is because the number of ions having an absorption radius is very small. Thus the application of equation (13) is not greatly in error. Moreover, similar charging equation has extensively been used by several authors in the nonlinear analysis even in presence of particle trapping (e.g. see reference [28,38,39] Sk E1Labany).

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